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**COVER NOTE**

| | |
|---|---|
| From: | European Commission |
| date of receipt: | 9 January 2015 |
| To: | General Secretariat of the Council |
| Subject: | Annex to Commission Directive ../.../EU of XXX establishing common noise assessment methods according to Directive 2002/49/EC |

Delegations will find attached document D034332/03 - Part 2.

Encl.: D034332/03 - Part 2

## APPENDIX A: DATA REQUIREMENTS

Section 2.7.6 of the main text describes in general terms the requirements for case-specific data describing an airport and its operations that are needed for noise contour calculations. The following datasheets are filled with example data for a hypothetical airport. Specific data formats will generally depend on the requirements and needs for the particular noise modelling system as well as the study scenario.

Note: It is recommended that geographic information (reference points etc.) be specified in Cartesian co-ordinates. The choice of the particular co-ordinate system usually depends on the maps available.

### A1 General airport data

| | | |
|---|---|---|
| **Aerodrome designation** | Hypothetical Airport | |
| **Co-ordinate system** | UTM, Zone 15, Datum WGS-84 | |
| **Aerodrome reference point, ARP** | 3 600 000 m E | 6 300 000 m N |
| | Mid-point of runway 09L-27R | |
| **Altitude of ARP** | 120 m / | |
| **Average air temperature at ARP*** | 12.0 ºC | |
| **Average relative humidity at ARP*** | 60 % | |
| **Average wind speed & direction*** | 5 kt | 270 degrees |
| **Source of topographical data** | Unknown | |

* Repeat for each time interval of interest (time of day, season etc.)

### A2 Runway description

| | | |
|---|---|---|
| **Runway designation** | 09L | |
| **Start of runway** | 3 599 000 m E | 6 302 000 m N |
| **End of runway** | 3 603 000 m E | 6 302 000 m N |

| | | |
|---|---|---|
| **Start of roll** | 3 599 000 m E | 6 302 000 m N |
| **Landing threshold** | 3 599 700 m E | 6 302 000 m N |
| **Altitude of start of runway** | 110 m | |
| **Mean runway gradient** | 0.001 | |

For displaced thresholds, runway description may be repeated or displaced thresholds can be described in the ground track description section.

## A3 Ground track description

In the absence of radar data the following information is needed to describe particular ground tracks.

| | |
|---|---|
| **Track No.** | 001 |
| **Track designation** | Dep 01 – 09L |
| **From runway** | 09L |
| **Type of track** | Departure |
| **Displacement from start of roll** | 0 m |
| **Number of subtracks:** | 7 |

**Backbone track description**

| Segment no. | Straight [m] | Curve | | | Standard deviation for lateral dispersion at segment end [m] |
|---|---|---|---|---|---|
| | | L/R | Heading change [°] | Radius [m] | |
| 1 | 10000 | | | | 2000 |
| 3 | | R | 90.00 | 3000 | 2500 |
| 4 | 20000 | | | | 3000 |

| | |
|---|---|
| **Track No.** | 002 |
| **Track designation** | App 01 – 09L – Disp 300 |
| **From runway** | 09L |

| | | | | | |
|---|---|---|---|---|---|
| **Type of track** | | | Approach | | |
| **Displacement from landing threshold** | | | 300 m | | |
| **Number of subtracks:** | | | 1 | | |
| **Backbone track description** | | | | | |
| Segment no. | Straight [m] | Curve | | | Standard deviation for lateral dispersion at segment end [m] |
| | | L/R | Heading change [°] | Radius [m] | |
| 1 | 30000 | | | | 0 |
| **Approach track information** | | | | | |
| **Glide angle for approach tracks** | | | 2.7° | | |
| **Flight altitude at glide slope interception** | | | 4000 ft | | |

## A4 Air traffic description

| | | |
|---|---|---|
| **Reference time period** | 366 d<br>(01-01-2014 to 31-12-2014) | = 8748 h |
| **Time of day period I** | From 7 to 19 h | = 12 h |
| **Time of day period II** | From 19 to 23 h | = 4 h |
| **Time of day period III** | From 23 to 7 h | = 8 h |

| | | | |
|---|---|---|---|
| **AIR TRAFFIC DESCRIPTION DATA SHEET – MOVEMENTS PER TRACK** | | | |
| **Ground track no.** | 001 | | |
| **Track designation** | Dep 01 – 09L | | |
| **Aircraft designation** | **Movements during time period** | | |
| | **I** | **II** | **III** |
| A/C 1, Dep.1 | 20000 | 4000 | 1000 |

| | | | |
|---|---|---|---|
| A/C 2, Dep.4 | 10000 | 5000 | 500 |
| A/C 4, Dep.3 | 2000 | 300 | 0 |

| | |
|---|---|
| **Ground track no.** | 002 |
| **Track designation** | Dep 01 – 09L – Disp 300 |

| **Aircraft designation** | **Movements during time period** | | |
|---|---|---|---|
| | **I** | **II** | **III** |
| A/C 1, App.1 | 18000 | 2000 | 5000 |
| A/C 2, App.1 | 10000 | 3000 | 2500 |
| A/C 4, App.1 | 1300 | 0 | 1000 |

## A5 Flight procedure data sheet

Example aircraft for a Chapter 3 Boeing 727-200 as derived from radar using the guidance set out in section 2.7.9 of the main text.

| | |
|---|---|
| Aircraft designation | B727C3 |
| NPD-Identifier from ANP database | JT8E5 |
| No. of engines | 3 |
| Mode of operation | Departure |
| Actual aircraft mass [t] | 71.5 |
| Headwind [m/s] | 5 |
| Temperature [°C] | 20 |
| Airport elevation [m] | 83 |

| Segment | Dist. from RP[1] | Height | Ground speed | Engine Power |
|---|---|---|---|---|

[1] The reference point RP is the start of roll for departures and the landing threshold for approaches.

| No. | [m] | [m] | [m/s] | [ [2]] |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 14568 |
| 2 | 2500 | 0 | 83 | 13335 |
| 3 | 3000 | 117 | 88 | 13120 |
| 4 | 4000 | 279 | 90 | 13134 |
| 5 | 4500 | 356 | 90 | 13147 |
| 6 | 5000 | 431 | 90 | 13076 |
| 7 | 6000 | 543 | 90 | 13021 |
| 8 | 7000 | 632 | 93 | 12454 |
| 9 | 8000 | 715 | 95 | 10837 |
| 10 | 10000 | 866 | 97 | 10405 |
| 11 | 12000 | 990 | 102 | 10460 |
| 12 | 14000 | 1122 | 111 | 10485 |
| 13 | 16000 | 1272 | 119 | 10637 |
| 14 | 18000 | 1425 | 125 | 10877 |
| 15 | 20000 | 1581 | 130 | 10870 |
| 16 | 25000 | 1946 | 134 | 10842 |
| 17 | 30000 | 2242 | 142 | 10763 |

Example for a procedural profile based on A/C-data stored in ANP database:

| | |
|---|---|
| Aircraft designation from ANP database | B727C3 |
| NPD-Identifier from ANP database | JT8E5 |
| No. of engines | 3 |
| Mode of operation | Departure |
| Actual aircraft mass [t] | 71.5 |

[2] Units corresponding to units in ANP database

| | | | | |
|---|---|---|---|---|
| Headwind [m/s] | | 5 | | |
| Temperature [°C] | | 15 | | |
| Airport elevation [m] | | 100 | | |
| | | | | |
| Segment No. | Mode | Target | Flaps | Engine Power |
| 1 | Takeoff | | 5 | Takeoff |
| 2 | Initial Climb | Altitude 1500 ft | 5 | Takeoff |
| 3 | Retract Flaps | 210 kts IAS<br>ROC 750 ft/min | 0 | Max Climb |
| 4 | Accelerate | 250 kts IAS<br>ROC 1500 ft/min | 0 | Max Climb |
| 5 | Climb | 10000 ft | 0 | Max Climb |

## APPENDIX B: FLIGHT PERFORMANCE CALCULATIONS

### Terms and symbols

The terms and symbols used in this appendix are consistent with those conventionally used by aircraft performance engineers. Some basic terms are explained briefly below for the benefit of users not familiar with them. To minimise conflict with the main body of the method, symbols are mostly defined separately within this appendix. Quantities that are referenced in the main body of the method are assigned common symbols; a few that are used differently in this appendix are marked with an asterix (*). There is some juxtaposition of US and SI units; again this is to preserve conventions that are familiar to users from different disciplines.

### Terms

| | |
|---|---|
| Break point | See Flat Rating |
| Calibrated airspeed | (Otherwise termed equivalent or indicated airspeed.) The speed of the aircraft relative to the air as indicated by a calibrated instrument on the aircraft. The true airspeed, which is normally greater, can be calculated from the calibrated airspeed knowing the air density. |
| Corrected net thrust | Net thrust is the propulsive force exerted by an engine on the airframe. At a given power setting (*EPR* or $N_1$) this falls with air density as altitude increases; corrected net thrust is the thrust at sea level. |
| Flat rating | For specific maximum component temperatures, the engine thrust falls as the ambient air temperature rises - and *vice-versa*. This means that there is a critical air temperature above which the *rated thrust* cannot be achieved. For most modern engines this is called the ‘flat rated temperature’ because, at lower air temperatures the thrust is automatically limited to the rated thrust to maximise service life. The thrust falls anyway at temperatures above the flat rated temperature - which is often called the *break point* or *break temperature*. |
| Speed | Magnitude of aircraft velocity vector (relative to aerodrome coordinate system) |
| Rated thrust | The service life of an aircraft engine is very dependent upon the operating temperatures of its components. The greater the power or trust generated, the higher the temperatures and the shorter the life. To balance performance and life requirements flat rated engines are assigned *thrust ratings* for take-off, climb and cruise which define normal maximum power settings. |
| Thrust setting parameter | The pilot cannot select a particular engine thrust; rather s/he chooses an appropriate setting of this parameter which is displayed in the cockpit. It is usually either the engine pressure ratio (*EPR*) or low- |

pressure rotor (or fan) rotational speed ($N_1$).

**Symbols**

Quantities are dimensionless unless otherwise stated. Symbols and abbreviations not listed below are used only locally and defined in the text. Subscripts 1 and 2 denote conditions at the start and end of a segment respectively. Overbars denote segment mean values, i.e. average of start and end values.

| | |
|---|---|
| $a$ | Average acceleration, ft/s$^2$ |
| $a_{max}$ | Maximum acceleration available, ft/s$^2$ |
| $A, B, C, D$ | Flap coefficients |
| $E, F, G_{A,B}, H$ | Engine thrust coefficients |
| $F_n$ | Net thrust per engine, lbf |
| $F_n/\delta$ | Corrected net thrust per engine, lbf |
| $G$ | Climb gradient |
| $G'$ | Engine-out climb gradient |
| $G_R$ | Mean runway gradient, positive uphill |
| $g$ | Gravitational acceleration, ft/s$^2$ |
| *ISA* | International Standard Atmosphere |
| $N$ * | No of engines supplying thrust |
| $R$ | Drag-to-lift ratio $C_D/C_L$ |
| *ROC* | Segment rate of climb (ft/min) |
| $s$ | Ground distance covered along ground track, ft |
| $s_{TO8}$ | Take-off distance into an 8 kt headwind, ft |
| $s_{TOG}$ | Take-off distance corrected for $w$ and $G_R$, ft |
| $s_{TOw}$ | Take-off distance into headwind $w$, ft |
| $T$ | Air temperature, °C |
| $T_B$ | Breakpoint temperature, °C |

| | |
|---|---|
| $V$ | Groundspeed, kt |
| $V_C$ | Calibrated airpeed, kt |
| $V_T$ | True airspeed, kt |
| $W$ | Aeroplane weight, lb |
| $w$ | Headwind speed, kt |
| $\Delta s$ | Still air segment length projected onto ground track, ft |
| $\Delta s_w$ | Segment length ground projection corrected for headwind, ft |
| δ | $p/p_O$, the ratio of the ambient air pressure at the aeroplane to the standard air pressure at mean sea level: $p_O$ = 101.325 kPa (or 1013.25 mb) |
| ε | Bank angle, radians |
| γ | Climb/descent angle, radians |
| θ | $(T + 273.15)/(T_0 + 273.15)$ the ratio of the air temperature at altitude to the standard air temperature at mean sea level: $T_0$ = 15.0 °C |
| σ * | $\rho/\rho_0$ = Ratio of air density at altitude to mean sea level value (also, $\sigma = \delta/\theta$) |

## B1 Introduction

### Flight path synthesis

In the main, this appendix recommends procedures for calculating an aeroplane flight profile, based on specified aerodynamic and powerplant parameters, aircraft weight, atmospheric conditions, ground track and operating procedure (flight configuration, power setting, forward speed, vertical speed etc). The operating procedure is described by a set of *procedural steps* that prescribe how to fly the profile.

The flight profile, for takeoff or approach, is represented by a series of straight-line segments, the ends of which are termed *profile points*. It is calculated using aerodynamic and thrust equations containing numerous coefficients and constants which must be available for the specific combination of airframe and engine. This calculation process is described in the text as the process of flight path *synthesis*.

Apart from the aircraft performance parameters, which can be obtained from the ANP database, these equations require specification of (1) aeroplane gross weight, (2) the number of engines, (3) air temperature, (4) runway elevation, and (5) the procedural steps (expressed in terms of power settings, flap deflections, airspeed and, during acceleration, average rate-of-climb/descent) for each segment during takeoff and approach. Each segment is then classified as a ground roll, take-off or landing, constant speed climb, power cutback, accelerating climb with or without flap retraction, descent with or without deceleration and/or flap deployment, or final landing approach. The flight profile is built up step by step, the starting parameters for each segment being equal to those at the end of the preceding segment.

The aerodynamic-performance parameters in the ANP database are intended to yield a reasonably accurate representation of an aeroplane's actual flight path for the specified reference conditions (see **section 2.7.6 of the main text**). But the aerodynamic parameters and engine coefficients have been shown to be adequate for air temperatures up to 43 °C, aerodrome altitudes up to 4,000ft and across the range of weights specified in the ANP database. The equations thus permit the calculation of flight paths for other conditions; i.e. non-reference aeroplane weight, wind speed, air temperature, and runway elevation (air pressure), normally with sufficient accuracy for computing contours of average sound levels around an airport.

**Section B-4** explains how the effects of turning flight are taken into account for departures. This allows bank angle to be accounted for when calculating the effects of lateral directivity (installation effects). Also, during turning flight, climb gradients will generally be reduced depending in the radius of the turn and the speed of the aeroplane. (The effects of turns during the landing approach are more complex and are not covered at present. However these will rarely influence noise contours significantly.)

**Sections B-5** to **B-9** describe the recommended methodology for generating departure flight profiles, based on ANP database coefficients and procedural steps.

**Sections B-10** and **B-11** describe the methodology used to generate approach flight profiles, based on ANP database coefficients and flight procedures.

**Section B-12** provides worked examples of the calculations.

Separate sets of equations are provided to determine the net thrust produced by jet engines and propellers respectively. Unless noted otherwise, the equations for aerodynamic performance of an aeroplane apply equally to jet and propeller-powered aeroplanes.

Mathematical symbols used are defined at the beginning of this appendix and/or where they are first introduced. In all equations the units of coefficients and constants must of course be consistent with the units of the corresponding parameters and variables. For consistency with the ANP database, the conventions of aircraft performance engineering are followed in this appendix; distances and heights in feet (ft), speed in knots (kt), mass in pounds (lb), force in pounds-force (high-temperature corrected net thrust), and so on - even though some dimensions (e.g. atmospheric ones) are expressed in SI units. Modellers using other unit systems should be very careful to apply appropriate conversion factors when adopting the equations to their needs.

### Flight path analysis

In some modelling applications the flight path information is provided not as procedural steps but as coordinates in position and time, usually determined by analysis of radar data. This is discussed in **section 2.7.7** of the main text. In this case the equations presented in this Appendix are used 'in reverse'; the engine thrust parameters are derived from the aircraft motion rather than vice-versa. In general, once the flight path data has been averaged and reduced to segment form, each segment being classified by climb or descent, acceleration or deceleration, and thrust and flap changes, this is relatively straightforward by comparison with synthesis which often involves iterative processes.

## B2 Engine thrust

The propulsive force produced by each engine is one of five quantities that need to be defined at the ends of each flight path segment (the others being height, speed, power setting and bank angle). Net thrust represents the component of engine gross thrust that is available for propulsion. For aerodynamic and acoustical calculations, the net thrust is referred to standard air pressure at mean sea level. This is known as *corrected net thrust*, $F_n/\delta$.

This will be either the net thrust available when operating at a specified *thrust rating*, or the net thrust that results when the *thrust-setting parameter* is set to a particular value. For a turbojet or turbofan engine operating at a specific thrust rating, corrected net thrust is given by the equation

$$F_n / \delta = E + F \cdot V_C + G_A \cdot h + G_B \cdot h^2 + H \cdot T \qquad \text{(B-1)}$$

where

| | |
|---|---|
| $F_n$ | is the net thrust per engine, lbf |
| $\delta$ | is the ratio of the ambient air pressure at the aeroplane to the standard air pressure at mean sea level, i.e., to 101.325 kPa (or |

| | |
|---|---|
| | 1013.25 mb) [ref. 1] |
| $F_n/\delta$ | is the corrected net thrust per engine, lbf |
| $V_C$ | is the calibrated airspeed, kt |
| $T$ | is the ambient air temperature in which the aeroplane is operating, °C, and |
| $E$, $F$, $G_A$, $G_B$, $H$ | are engine thrust constants or coefficients for temperatures below the engine flat rating temperature at the thrust rating in use (on the current segment of the takeoff/climbout or approach flight path), lb.s/ft, lb/ft, lb/ft$^2$, lb/°C. Obtainable from the ANP database. |

Data are also provided in the ANP database to allow calculation of non-rated thrust as a function of a thrust setting parameter. This is defined by some manufacturers as engine pressure ratio *EPR*, and by others as low-pressure rotor speed, or fan speed, $N_1$. When that parameter is *EPR*, Equation B-1 is replaced by

$$F_n/\delta = E + F \cdot V_C + G_A \cdot h + G_B \cdot h^2 + H \cdot T + K_1 \cdot EPR + K_2 \cdot EPR^2 \qquad \text{(B-2)}$$

where $K_1$ and $K_2$ are coefficients, from the ANP database that relate corrected net thrust and engine pressure ratio in the vicinity of the engine pressure ratio of interest for the specified aeroplane Mach number.

When engine rotational speed $N_1$ is the parameter used by the cockpit crew to set thrust, the generalised thrust equation becomes

$$F_n/\delta = E + F \cdot V_C + G_A \cdot h + G_B \cdot h^2 + H \cdot T + K_3 \cdot \left(\frac{N_1}{\sqrt{\theta}}\right) + K_4 \cdot \left(\frac{N_1}{\sqrt{\theta}}\right)^2 \qquad \text{(B-3)}$$

where

| | |
|---|---|
| $N_1$ | is the rotational speed of the engine's low-pressure compressor (or fan) and turbine stages, % |
| $\theta$ | = (T + 273)/288.15, the ratio of the absolute total temperature at the engine inlet to the absolute standard air temperature at mean sea level [ref. 1]. |
| $\frac{N_1}{\sqrt{\theta}}$ | is the corrected low pressure rotor speed, %; and |
| $K_3$, $K_4$ | are constants derived from installed engine data encompassing the $N_1$ speeds of interest. |

Note that for a particular aeroplane $E, F, G_A, G_B$ and $H$ in equations B-2 and B-3 might have different values from those in equation B-1.

Not every term in the equation will always be significant. For example, for flat-rated engines operating in air temperatures below the break point (typically 30°C), the temperature term may not be required. For engines not flat rated, ambient temperature must be considered when designating rated thrust. Above the engine flat rating temperature, a different set of engine thrust coefficients $(E, F, G_A, G_B$ and $H)_{high}$ must be used to determine the thrust level available. Normal practice would then be to compute $F_n/\delta$ using both the low temperature and high temperature coefficients and to use the higher thrust level for temperatures *below* the flat rating temperature and use the lower calculated thrust level for temperature *above* the flat rating temperature.

Where only low temperature thrust coefficients are available, the following relationship may be used:

$$(F_n/\delta)_{high} = F \cdot V_C + (E + H \cdot T_B) \cdot (1 - 0.006 \cdot T)/(1 - 0.006 \cdot T_B) \qquad \text{(B-4)}$$

where

| | |
|---|---|
| $(F_n/\delta)_{high}$ | high-temperature corrected net thrust (lbf), |
| $T_B$ | breakpoint temperature (in the absence of a definitive value assume a default value of 30 °C). |

The ANP database provides values for the constants and coefficients in equations B-1 to B-4.

For propeller driven aeroplanes, corrected net thrust per engine should be read from graphs or calculated using the equation

$$F_n/\delta = (326 \cdot \eta \cdot P_p / V_T)/\delta \qquad \text{(B-5)}$$

where

| | |
|---|---|
| $\eta$ | is the propeller efficiency for a particular propeller installation and is a function of propeller rotational speed and aeroplane flight speed |
| $V_T$ | is the true airspeed, kt |
| $P_p$ | is net propulsive power for the given flight condition, e.g. max takeoff or max climb power, hp |

Parameters in Equation B-5 are provided in the ANP database for maximum takeoff thrust and maximum climb thrust settings.

True airspeed $V_T$ is estimated from the calibrated airspeed $V_C$ using the relationship

$$V_T = V_C / \sqrt{\sigma} \qquad \text{(B-6)}$$

where σ is the ratio of the air density at the aeroplane to the mean sea-level value.

**Guidance on operation with reduced takeoff thrust**

Often, aircraft takeoff weights are below maximum allowable and/or the available runway field length exceeds the minimum required with the use of maximum takeoff thrust. In these cases, it is common practice to reduce engine thrust below maximum levels in order to prolong engine life and, sometimes, for noise abatement purposes. Engine thrust can only be reduced to levels that maintain a required margin of safety. The calculation procedure used by airline operators to determine the amount of thrust reduction is regulated accordingly: it is complex and takes into account numerous factors including takeoff weight, ambient air temperature, declared runway distances, runway elevation and runway obstacle clearance criteria. Therefore the amount of thrust reduction varies from flight to flight.

As they can have a profound effect upon departure noise contours, modellers should take reasonable account of reduced thrust operations and, to make best possible provision, to seek practical advice from operators.

If such advice is not available it is still advisable to make some allowance by alternative means. It is impractical to mirror the operators' calculations for noise modelling purposes; nor would they be appropriate alongside the conventional simplifications and approximations which are made for the purposes of calculating long term average noise levels. As a practicable alternative the following guidance is provided. It should be emphasised that considerable research is ongoing in this area and thus, this guidance is subject to change.

Analysis of FDR data has shown that the level of thrust reduction is strongly correlated with ratio of the actual takeoff weight to the Regulated Takeoff Weight (RTOW), down to a fixed lower limit[3]; i.e.

$$F_n / \delta = \left(F_n / \delta\right)_{max} \cdot W / W_{RTOW} \qquad \text{(B-7)}$$

where $(F_n/\delta)_{max}$ is the maximum rated thrust, $W$ is the actual gross take-off weight and $W_{RTOW}$ is the Regulated Takeoff Weight.

The RTOW is the maximum takeoff weight that can be safely used, whilst satisfying takeoff field length, engine-out and obstacle requirements. It is a function of the available runway length, airfield elevation, temperature, headwind, and flap angle. This information can be obtained from operators and should be more readily available than data on actual levels of reduced thrust. Alternatively, it may be computed using data contained in aircraft flight manuals.

**Reduced Climb Thrust**

[3] Airworthiness authorities normally stipulate a lower thrust limit, often 25 percent below maximum.

When employing reduced take-off thrust, operators often, but not always, reduce climb thrust from below maximum levels[4]. This prevents situations occurring where, at the end of the initial climb at take-off thrust, power has to be increased rather than cut back. However, it is more difficult to establish a rationale for a common basis here. Some operators use fixed detents below maximum climb thrust, sometimes referred to as Climb 1 and Climb 2, typically reducing climb thrust by 10 and 20 percent respectively relative to maximum. It is recommended that whenever reduced takeoff thrust is used, climb thrust levels also be reduced by 10 percent.

## B3 Vertical profiles of air temperature, pressure, density and windspeed

For the purposes of this document, the variations of temperature, pressure and density with height above mean sea level are taken to be those of the International Standard Atmosphere. The methodologies described below have been validated for aerodrome altitudes up to 4000 ft above sea level and for air temperatures up to 43 °C (109 °F).

Although, in reality, mean wind velocity varies with both height and time, it is not usually practicable to take account of this for noise contour modelling purposes. Instead, the flight performance equations given below are based on the common assumption that the aeroplane is heading directly into a (default) headwind of 8 kt at all times - regardless of compass bearing (although no explicit account of mean wind velocity is taken in sound propagation calculations). Methods for adjusting the results for other headwind speeds are provided.

## B4 The effects of turns

The remainder of this appendix explains how to calculate the required properties of the segments joining the profile points *s,z* that define the two-dimensional flight path in the vertical plane above the ground track. Segments are defined in sequence in the direction of motion. At the end of any one segment (or at the start of roll in the case of the first for a departure) where the operational parameters and the next procedural step are defined, the need is to calculate the climb angle and track distance to the point where the required height and/or speed are reached.

If the track is straight, this will be covered by a single profile segment, the geometry of which can then be determined directly (albeit sometimes with a degree of iteration). But if a turn starts or ends, or changes in radius or direction, before the required end-conditions are reached, a single segment would be insufficient because the aircraft lift and drag change with bank angle. <u>To account for the effects of the turn on the climb</u>, additional profile segments are required to implement the procedural step - as follows.

The construction of the ground track is described in section **2.7.13** of the text. This is done independently of any aircraft flight profile (although with care not to define turns that could not be flown under normal operating constraints). But as the flight profile - height and speed

[4] To which thrust is reduced after the initial climb at take-off power.

as a function of track distance - is affected by turns so that the flight profile cannot be determined independently of the ground track.

To maintain speed in a turn the aerodynamic wing lift has to be increased, to balance centrifugal force as well as the aircraft weight. This in turn increases drag and, consequently the propulsive thrust required. The effects of the turn are expressed in the performance equations as functions of bank angle ε which, for an aircraft in level flight turning at constant speed on a circular path, is given by

$$\varepsilon = \tan^{-1}\left\{\frac{2.85 \cdot V^2}{r \cdot g}\right\} \tag{B-8}$$

| | | |
|---|---|---|
| where | $V$ | is the groundspeed, kt |
| | $r$ | is the turn radius, ft |
| and | g | is the acceleration due to gravity, ft/s$^2$ |

All turns are assumed to have a constant radius and second-order effects associated with non-level flight paths are disregarded; bank angles are based on the turn radius *r* of the ground track only.

To implement a procedural step a provisional profile segment is first calculated using the bank angle ε at the start point - as defined by Equation B-8 for the track segment radius *r*. If the calculated length of the provisional segment is such that it does not cross the start or end of a turn, the provisional segment is confirmed and attention turns to the next step.

But if the provisional segment crosses one or more starts or ends of turns (where ε changes)[5], the flight parameters at the first such point are estimated by interpolation (see section **2.7.13**), saved along with its coordinates as end-point values, and the segment truncated. The second part of the procedural step is then applied from that point - once more assuming provisionally that it can be completed in a single segment with the same end conditions but with the new start point and new bank angle. If this second segment then passes another change of turn radius/direction, a third segment will be required - and so on until the end-conditions are achieved.

**Approximate method**

It will be apparent that accounting fully for the effects of turns, as described above, involves considerable computational complexity because the climb profile of any aircraft has to be calculated separately for each ground track that it follows. But changes to the vertical profile caused by turns usually have a markedly smaller influence on the contours than the changes of bank angle, and some users may prefer to avoid the complexity - at the cost of some loss of precision - by disregarding the effects of turns on profiles while still accounting for the bank angle in the calculation of lateral sound emission (see section 2.7.19). Under this

[5] To avoid contour discontinuities caused by instantaneous changes of bank angle at the junctions between straight and turning flight, sub-segments are introduced into the noise calculations to allow linear transitions of bank angle over the first and last 5° of the turn. These are not necessary in the performance calculations; the bank angle is always given by equation B-8.

approximation profile points for a particular aircraft operation are calculated once only, assuming a straight ground track (for which ε = 0).

## B5 Takeoff ground roll

Take-off thrust accelerates the aeroplane along the runway until lift-off. Calibrated airspeed is then assumed to be constant throughout the initial part of the climbout. Landing gear, if retractable, is assumed to be retracted shortly after lift-off.

For the purpose of this document, the actual takeoff ground-roll is approximated by an equivalent take-off distance (into a default headwind of 8 kt), $s_{TO8}$, defined as shown in **Figure B-1**, as the distance along the runway from brake release to the point where a straight line extension of the initial landing-gear-retracted climb flight path intersects the runway.

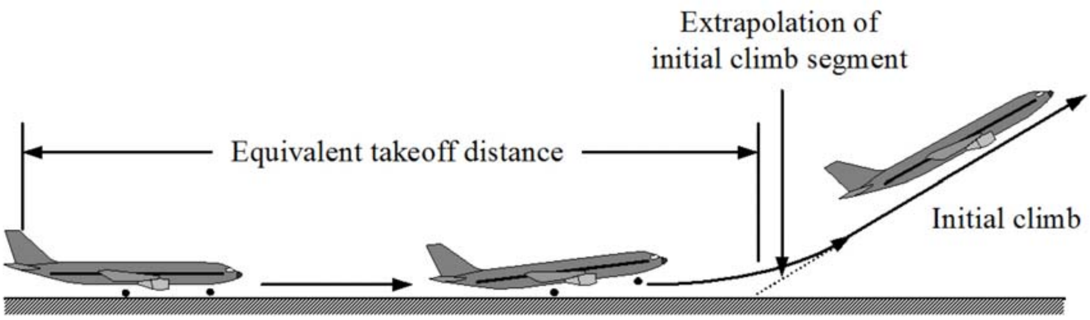


**Figure B-1: Equivalent takeoff distance**

On a level runway, the equivalent takeoff ground-roll distance $s_{TO8}$ in feet is determined from

$$s_{TO8} = \frac{B_8 \cdot \theta \cdot (W/\delta)^2}{N \cdot (F_n/\delta)} \qquad \text{(B-9)}$$

where

| | |
|---|---|
| $B_8$ | is a coefficient appropriate to a specific aeroplane/flap-deflection combination for the ISA reference conditions, including the 8-knot headwind, ft/lbf |
| $W$ | is the aeroplane gross weight at brake release, lbf |
| $N$ | is the number of engines supplying thrust. |

Note: Since equation B-9 accounts for variation of thrust with airspeed and runway elevation, for a given aeroplane the coefficient $B_8$ depends only on flap deflection.

For headwind other than the default 8kt, the takeoff ground-roll distance is corrected by using:

$$s_{TOw} = s_{TO8} \cdot \frac{(V_C - w)^2}{(V_C - 8)^2} \qquad \text{(B-10)}$$

where

| | |
|---|---|
| $S_{TOw}$ | is the ground-roll distance corrected for headwind *w*, ft |
| $V_C$ | (in this equation) is the calibrated speed at takeoff rotation, kt |
| $w$ | is the headwind, kt |

The takeoff ground-roll distance is also corrected for runway gradient as follows:

$$s_{TOG} = s_{TOw} \cdot \frac{a}{(a - g \cdot G_R)} \qquad \text{(B-11)}$$

where

| | |
|---|---|
| $S_{TOG}$ | is the ground-roll distance (ft) corrected for headwind and runway gradient, |
| $a$ | is the average acceleration along the runway, equal to $\left(V_C \cdot \sqrt{\sigma}\right)^2 / (2 \cdot s_{TOw})$, ft/s$^2$ |
| $G_R$ | is the runway gradient; positive when taking-off uphill |

## B6 Climb at constant speed

This type of segment is defined by the aeroplane's calibrated airspeed, flap setting, and the height and bank angle at its end, together with the headwind speed (default 8 kt). As for any segment, the segment start parameters including corrected net thrust are put equal to those at the end of the preceding segment - there are no discontinuities (except of flap angle and bank angle which, in these calculations, are allowed to change in steps). The net thrusts at the segment end are first calculated using the appropriate equation from B-1 to B-5. The average geometric climb angle γ (see **Figure B-1**) is then given by

$$\gamma = \arcsin\left(K \cdot \left[N \cdot \frac{\overline{F_n / \delta}}{\overline{W / \delta}} - \frac{R}{\cos\varepsilon}\right]\right) \qquad \text{(B-12)}$$

where the over-bars denote mid-segment values (= average of start-point and end-point values - generally the mid-segment values) and

| | |
|---|---|
| $K$ | is a speed-dependent constant equal to 1.01 when $V_C \leq 200$ kt or 0.95 |

otherwise. This constant accounts for the effects on climb gradient of climbing into an 8-knot headwind and the acceleration inherent in climbing at constant calibrated airspeed (true speed increases as air density diminishes with height).

*R* is the ratio of the aeroplane's drag coefficient to its lift coefficient appropriate to the given flap setting. The landing gear is assumed to be retracted.

$\varepsilon$ Bank angle, radians

The climb angle is corrected for headwind *w* using:

$$\gamma_w = \gamma \cdot \frac{(V_C - 8)}{(V_C - w)} \qquad \text{(B-13)}$$

where $\gamma_w$ is the average climb angle corrected for headwind.

The distance that the aeroplane traverses along the ground track, $\Delta s$, while climbing at angle $\gamma_w$, from an initial altitude $h_1$ to a final altitude $h_2$ is given by

$$\Delta s = \frac{(h_2 - h_1)}{\tan \gamma_w} \qquad \text{(B-14)}$$

As a rule, two distinct phases of a departure profile involve climb at constant airspeed. The first, sometime referred to as the *initial climb segment* is immediately after lift-off, where safety requirements dictate that the aeroplane is flown at a minimum airspeed of least the takeoff safety speed. This is a regulated speed and should be achieved by 35ft above the runway during normal operation. However, it is common practice to maintain a initial climb speed slightly beyond the takeoff safety speed, usually by 10-20kt, as this tends to improve the initial climb gradient achieved. The second is after flap retraction and initial acceleration, referred to as *continuing climb*.

During the initial climb, the airspeed is dependent on the takeoff flap setting and the aeroplane gross weight. The calibrated initial climb speed $V_{CTO}$ is calculated using the first order approximation:

$$V_{CTO} = C \cdot \sqrt{W} \qquad \text{(B-15)}$$

where *C* is a coefficient appropriate to the flap setting (kt/√lbf), read from the ANP database.

For continuing climb after acceleration, the calibrated airspeed is a user input parameter.

## B7 Power cutback (transition segment)

Power is reduced, or *cut back*, from take-off setting at some point after takeoff in order to extend engine life and often to reduce noise in certain areas. Thrust is normally cut back during either a constant speed climb segment (**Section B6)** or an acceleration segment

(**Section B8)**. As it is a relatively brief process, typically of only 3 - 5 seconds duration, is it modelled by adding a 'transition segment' to the primary segment. This is usually taken to cover a horizontal ground distance of 1000 ft (305 m).

**Amount of thrust reduction**

In normal operation the engine thrust is reduced to the maximum climb thrust setting. Unlike the take-off thrust, climb thrust can be sustained indefinitely, usually in practice until the aeroplane has reached its initial cruise altitude. The maximum climb thrust level is determined with equation B-1 using the manufacturer supplied maximum thrust coefficients. However, noise abatement requirements may call for additional thrust reduction, sometimes referred to as a deep cutback. For safety purposes the maximum thrust reduction is limited[6] to an amount determined by the performance of the aeroplane and the number of engines.

The minimum "reduced-thrust" level is sometimes referred to as the engine-out "reduced thrust":

$$(F_n/\delta)_{engine.out} = \frac{(W/\delta_2)}{(N-1)} \cdot \left[ \frac{\sin(\arctan(0.01 \cdot G'))}{K} + \frac{R}{\cos\varepsilon} \right] \tag{B-16}$$

where

$\delta_2$ is the pressure ratio at altitude $h_2$

$G'$ is the engine-out percentage climb gradient:

= 0% for aeroplanes with automatic thrust restoration systems; otherwise,

= 1.2% for 2-engine aeroplane

= 1.5% for 3-engine aeroplane

= 1.7% for 4-engine aeroplane

**Constant speed climb segment with cutback**

The climb segment gradient is calculated using equation B-12, with thrust calculated using either B-1 with maximum climb coefficients, or B-16 for reduced thrust. The climb segment is then broken into two sub-segments, both having the same climb angle. This is illustrated in **Figure B-2**.

6 "Noise Abatement Procedures", ICAO Document 8168 "PANS-OPS" Vol.1 Part V, Chapter 3, ICAO 2004.

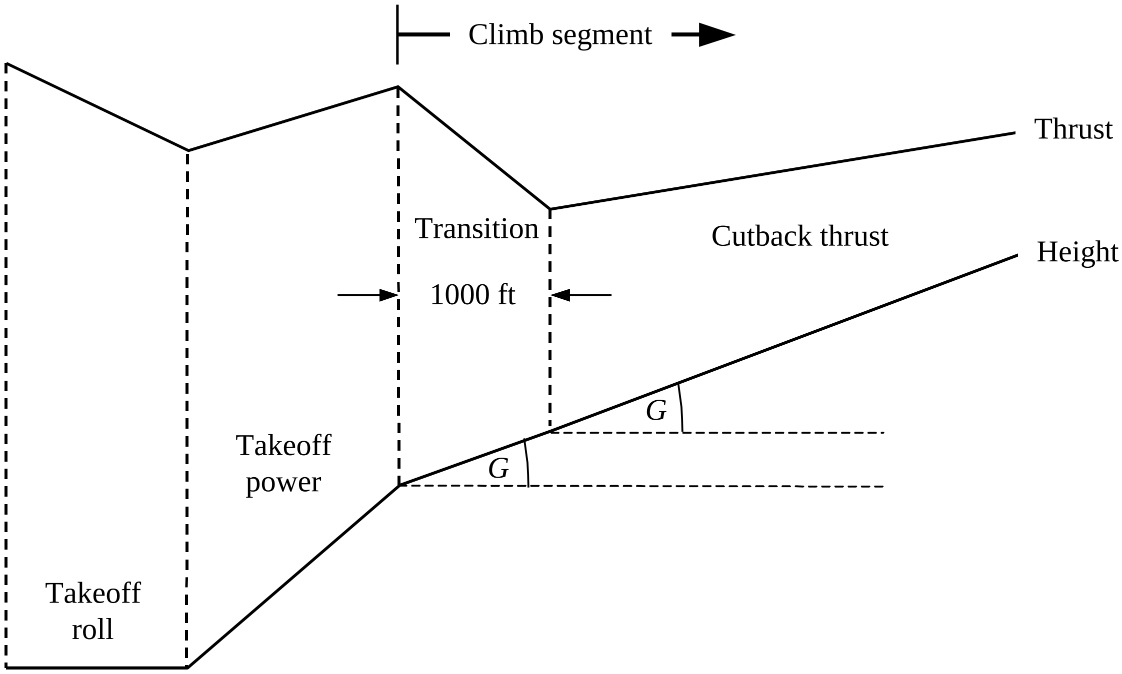


**Figure B-2: Constant speed climb segment with cutback (illustration – not to scale)**

The first sub-segment is assigned a 1000 ft (304 m) ground distance, and the corrected net thrust per engine at the end of 1000 ft is set equal to the cutback value. (If the original horizontal distance is less than 2000 ft, one half of the segment is used to cutback thrust.) The final thrust on the second sub-segment is also set equal to the cutback thrust. Thus, the second sub-segment is flown at constant thrust.

## B8 Accelerating climb and flap retraction

This usually follows the initial climb. As for all flight segments, the start-point altitude $h_1$, true airspeed $V_{T1}$, and thrust $(F_n/\delta)_1$ are those from the end of the preceding segment. The end-point calibrated airspeed $V_{C2}$ and the average climb rate *ROC* are user inputs (bank angle ε is a function of speed and radius of turn). As they are interdependent, the end altitude $h_2$, end true airspeed $V_{T2}$, end thrust $(F_n/\delta)_2$ and segment track length $\Delta s$ have to be calculated by iteration; the end altitude $h_2$ is guessed initially and then recalculated repeatedly using equations B-16 and B-17 until the difference between successive estimates is less than a specified tolerance, e.g. one foot. A practical initial estimate is $h_2 = h_1 + 250$ feet.

The segment track length (horizontal distance covered) is estimated as:

$$s_{seg} = 0.95 \cdot k^2 \cdot (V_{T2}^2 - V_{T1}^2) / 2(a_{max} - G \cdot g) \qquad \text{(B-17)}$$

where

| | |
|---|---|
| 0.95 | is a factor to account for effect of 8 kt headwind when climbing at 160 kt |
| $k$ | is a constant to convert knots to ft/sec = 1.688 ft/s per kt |
| $V_{T2}$ | = true airspeed at segment end, kt: $V_{T2} = V_{C2} / \sqrt{\sigma_2}$ |
| | where $\sigma_2$ = air density ratio at end altitude $h_2$ |
| $a_{max}$ | = maximum acceleration in level flight (ft/s$^2$) <br> $= g[\, N \cdot \overline{F_n / \delta} / (\overline{W / \delta}) - R / \cos\varepsilon \,]$ |
| $G$ | = climb gradient $\approx \dfrac{ROC}{60 \cdot k \cdot \overline{V_T}}$ |
| | where $ROC$ = climb rate, ft/min |

Using this estimate of $\Delta s$, the end altitude $h_2'$ is then re-estimated using:

$$h_2' = h_1 + s \cdot G / 0.95 \qquad \text{(B-18)}$$

As long as the error $|h_2' - h_2|$ is outside the specified tolerance, the steps B-17 and B-18 are repeated using the current iteration segment-end values of altitude $h_2$, true airspeed $V_{T2}$, corrected net thrust per engine $(F_n/\delta)_2$. When the error is within the tolerance, the iterative cycle is terminated and the acceleration segment is defined by the final segment-end values.

Note: If during the iteration process $(a_{max} - G \cdot g) < 0.02g$, the acceleration may be too small to achieve the desired $V_{C2}$ in a reasonable distance. In this case, the climb gradient can be limited to $G = a_{max}/g - 0.02$, in effect reducing the desired climb rate in order to maintain acceptable acceleration. If $G < 0.01$ it should be concluded there is not enough thrust to achieve the acceleration and climb rate specified; the calculation should be terminated and the procedure steps revised[7].

The acceleration segment length is corrected for headwind $w$ by using:

$$\Delta s_w = \Delta s \cdot \frac{(V_T - w)}{(V_T - 8)} \qquad \text{(B-19)}$$

**Accelerating segment with cutback**

Thrust cutback is inserted into an acceleration segments in the same way as for a constant speed segment; by turning its first part into a transition segment. The cutback thrust level is calculated as for the constant-speed cutback thrust procedure, using equation B-1 only. Note it is not generally possible to accelerate and climb whilst maintaining the minimum engine-out thrust setting. The thrust transition is assigned a 1000 ft (305 m) ground distance, and the corrected net thrust per engine at the end of 1000 ft is set equal to the cutback value. The

[7] In either case the computer model should be programmed to inform the user of the inconsistency.

speed at the end of the segment is determined by iteration for a segment length of 1000 ft. (If the original horizontal distance is less than 2000 ft, one half of the segment is used for thrust change.) The final thrust on the second sub-segment is also set equal to the cutback thrust. Thus, the second sub-segment is flown at constant thrust.

## B9 Additional climb and acceleration segments after flap retraction

If additional acceleration segments are included in the climbout flight path, equations B-12 to B-19 should be used again to calculate the ground-track distance, average climb angle, and height gain for each. As before, the final segment height must be estimated by iteration.

## B10 Descent and deceleration

Approach flight normally requires the aeroplane to descend and decelerate in preparation for the final approach segment where the aeroplane is configured with approach flap and gear down. The flight mechanics are unchanged from the departure case; the main difference is that the height and speed profile is generally known, and it is the engine thrust levels that must be estimated for each segment. The basic force balance equation is:

$$F_n / \delta = W \cdot \frac{R \cdot \cos\gamma + \sin\gamma + a/g}{N \cdot \delta} \qquad \text{(B-20)}$$

Equation B-20 may be used in two distinct ways. First the aeroplane speeds at the start and end of a segment may be defined, along with a descent angle (or level segment distance) and initial and final segment altitudes. In this case the deceleration may be calculated using:

$$a = \frac{(V_2/\cos\gamma)^2 - (V_1/\cos\gamma)^2}{(2 \cdot \Delta s/\cos\gamma)} \qquad \text{(B-21)}$$

where $\Delta s$ is the ground distance covered and $V_1$ and $V_2$ and are the initial and final groundspeeds calculated using

$$V = \frac{V_C \cdot \cos\gamma}{\sqrt{\sigma}} - w \qquad \text{(B-22)}$$

Equations B-20, B-21 and B-22 confirm that whilst decelerating over a specified distance at a constant rate of descent, a stronger headwind will result in more thrust being required to maintain the same deceleration, whilst a tailwind will require less thrust to maintain the same deceleration.

In practice most, if not all decelerations during approach flight are performed at idle thrust. Thus for the second application of Equation B-20, thrust is defined at an idle setting and the equation is solved iteratively to determine (1) the deceleration and (2) the height at the end of the deceleration segment - in a similar manner to the departure acceleration segments. In this case, deceleration distance can be very different with head and tail winds and it is sometimes necessary to reduce the descent angle in order to obtain reasonable results.

For most aeroplanes, idle thrust is not zero and, for many, it is also a function of flight speed. Thus, Equation B-20 is solved for the deceleration by inputting an idle thrust; the idle thrust is calculated using an equation of the form:

$$\left(F_n/\delta\right)_{idle} = E_{idle} + F_{idle}\cdot V_C + G_{A,idle}\cdot h + G_{B,idle}\cdot h^2 + H_{idle}\cdot T \qquad \text{(B-23)}$$

where ($E_{idle}$, $F_{idle,}$ $G_{A,idle}$, $G_{B,idle}$ and $H_{idle}$) are idle thrust engine coefficients available in the ANP database.

## B11 Landing approach

The landing approach calibrated airspeed, $V_{CA}$, is related to the landing gross weight by an equation of the same form as Equation B-11, namely

$$V_{CA} \approx D\cdot\sqrt{W} \qquad \text{(B-24)}$$

where the coefficient $D$ (kt/√lbf) corresponds to the landing flap setting.

The corrected net thrust per engine during descent along the approach glideslope is calculated by solving equation B-12 for the landing weight $W$ and a drag-to-lift ratio $R$ appropriate for the flap setting with landing gear extended. The flap setting should be that typically used in actual operations. During landing approach, the glideslope descent angle γ may be assumed constant. For jet-powered and multi-engine propeller aeroplanes, γ is typically −3°. For single-engine, propeller-powered aeroplanes, γ is typically −5°.

The average corrected net thrust is calculated by inverting equation B-12 using K=1.03 to account for the deceleration inherent in flying a descending flight path into an 8-knot reference headwind at the constant calibrated airspeed given by equation B-24, i.e.

$$\overline{F_n/\delta} = \frac{\overline{W/\delta}}{N}\cdot\left(R + \frac{\sin\gamma}{1.03}\right) \qquad \text{(B-25)}$$

For headwinds other than 8kt, average corrected net thrust becomes

$$\left(\overline{F_n/\delta}\right)_w = \overline{F_n/\delta} + 1.03\cdot\overline{W/\delta}\cdot\frac{\sin\gamma\cdot(w-8)}{N\cdot V_{CA}} \qquad \text{(B-26)}$$

The horizontal distance covered is calculated by:

$$\Delta s = \frac{(h_2 - h_1)}{\tan \gamma} \tag{B-27}$$

(positive since $h_1 > h_2$ and $\gamma$ is negative).

## APPENDIX C: MODELLING OF LATERAL GROUND TRACK SPREADING

It is recommended that, in the absence of radar data, lateral ground track dispersion be modelled on the assumption that the spread of tracks perpendicular to the backbone track follows a Gaussian normal distribution. Experience has shown that this assumption is a reasonable one in most cases.

Assuming a Gaussian distribution with a standard deviation *S*, illustrated in **Figure C-1**, about 98.8 percent of all movements fall within boundaries of $\pm 2.5 \cdot S$ (i.e. within a swathe of width of $5 \cdot S$).

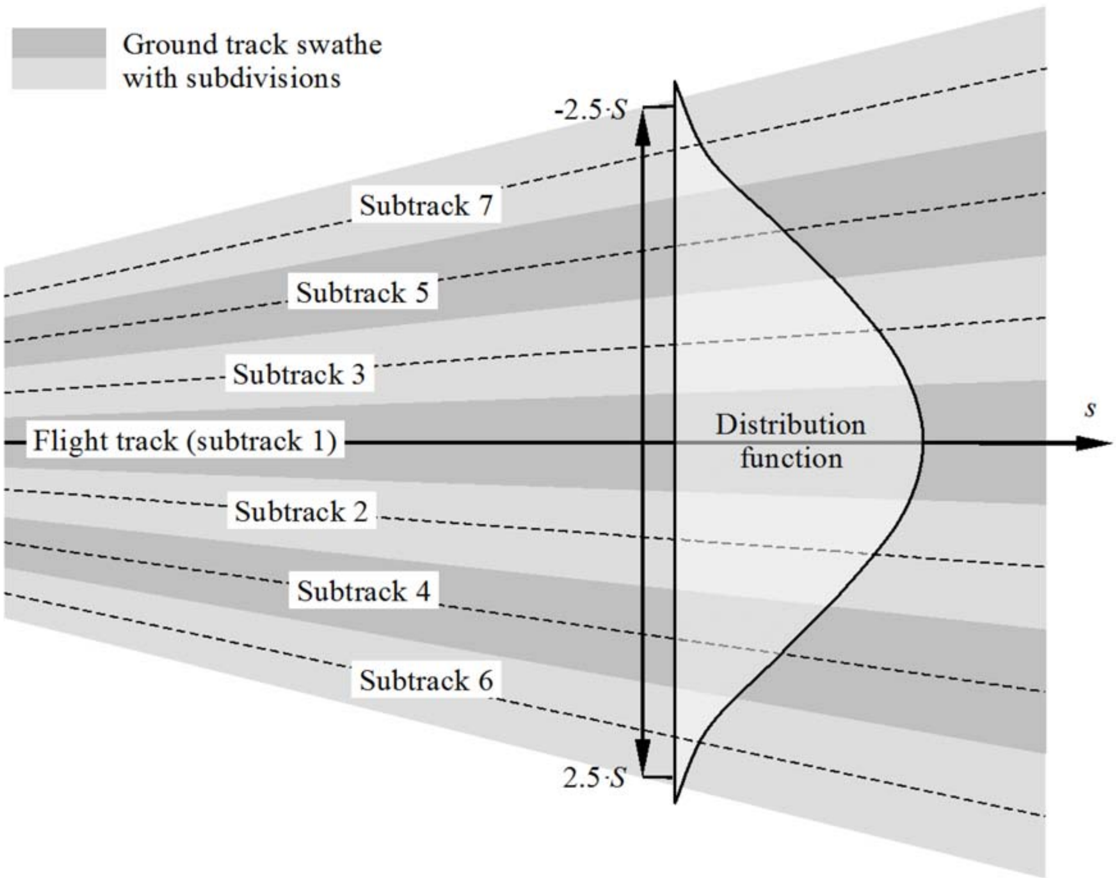


**Figure C-1: Subdivision of a ground track into 7 subtracks. The width of the swathe is 5 times the standard deviation of the ground track spreading.**

A Gaussian distribution can normally be modelled adequately using 7 discrete sub-tracks evenly spaced between the $\pm 2.5 \cdot S$ boundaries of the swathe as shown in **Figure C-1**.

However, the adequacy of the approximation depends on the relationship of the sub-track track separation to the heights of the aircraft above. There may be situations (very tight or very dispersed tracks) where a different number of subtracks is more appropriate. Too few subtracks cause ‘fingers’ to appear in the contour. **Tables C-1** and **C-2** show the parameters for a subdivision into between 5 and 13 subtracks. **Table C-1** shows the location of the particular subtracks, **Table C-2** the corresponding percentage of movements on each subtrack.

| Subtrack number | Location of subtracks for subdivision into | | | | |
|---|---|---|---|---|---|
| | **5 subtracks** | **7 subtracks** | **9 subtracks** | **11 subtracks** | **13 subtracks** |
| **12 / 13** | | | | | ±2.31·*S* |
| **10 / 11** | | | | ±2.27·*S* | ±1.92·*S* |
| **8 / 9** | | | ±2.22·*S* | ±1.82·*S* | ±1.54·*S* |
| **6 / 7** | | ±2.14·*S* | ±1.67·*S* | ±1.36·*S* | ±1.15·*S* |
| **4 / 5** | ±2.00·*S* | ±1.43·*S* | ±1.11·*S* | ±0.91·*S* | ±0.77·*S* |
| **2 / 3** | ±1.00·*S* | ±0.71·*S* | ±0.56·*S* | ±0.45·*S* | ±0.38·*S* |
| **1** | 0 | 0 | 0 | 0 | 0 |

**Table C-1: Location of 5, 7, 9, 11 or 13 subtracks. The overall width of the swathe (containing 98% of all movements) is 5 times the standard deviation**

| Subtrack number | Percentage of movements on subtrack for subdivision into | | | | |
|---|---|---|---|---|---|
| | **5 subtracks** | **7 subtracks** | **9 subtracks** | **11 subtracks** | **13 subtracks** |
| **12 / 13** | | | | | 1.1 % |
| **10 / 11** | | | | 1.4 % | 2.5 % |
| **8 / 9** | | | 2.0 % | 3.5 % | 4.7 % |
| **6 / 7** | | 3.1 % | 5.7 % | 7.1 % | 8.0 % |
| **4 / 5** | 6.3 % | 10.6 % | 12.1 % | 12.1 % | 11.5 % |
| **2 / 3** | 24.4 % | 22.2 % | 19.1 % | 16.6 % | 14.4 % |
| **1** | 38.6 % | 28.2 % | 22.2 % | 18.6 % | 15.6 % |

**Table C-2: Percentage of movements on 5, 7, 9, 11 or 13 subtracks. The overall width of the swathe (containing 98% of all movements) is 5 times the standard deviation**

## APPENDIX D: RECALCULATION OF NPD-DATA FOR NON-REFERENCE CONDITIONS

The noise level contributions from each segment of the flight path are derived from the NPD-data stored in the international ANP database. However it must be noted that these data have been normalised using average atmospheric attenuation rates defined in SAE AIR-1845.. Those rates are averages of values determined during aircraft noise certification testing in Europe and the USA. The wide variation of atmospheric conditions (temperature and relative humidity) in those tests is shown in **Figure D-1**.

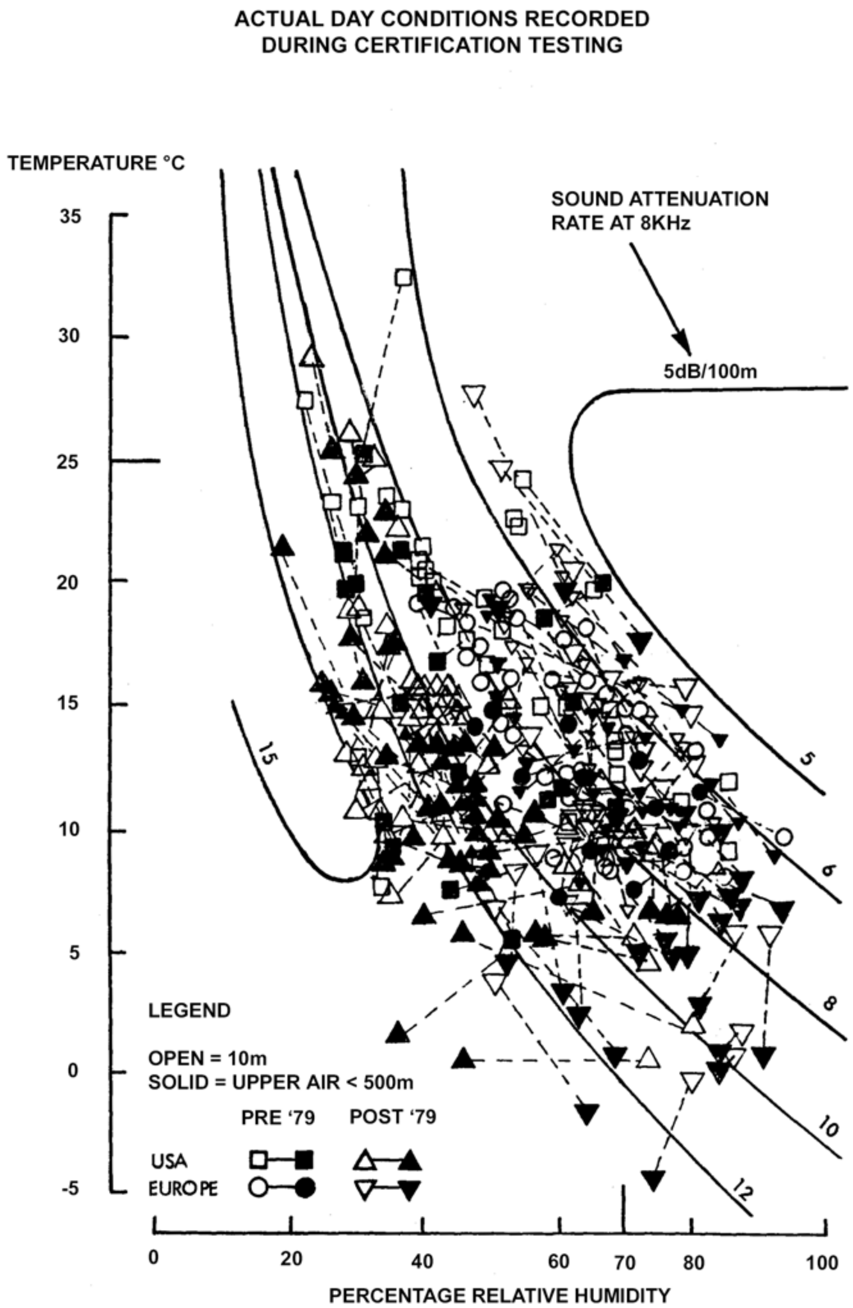


**Figure D-1: Meteorological conditions recorded during noise certification tests.**

The curves overlaid on **Figure D-1**, calculated using an industry standard atmospheric attenuation model ARP 866A, illustrate that across the test conditions a substantial variation of high frequency (8kHz) sound absorption would be expected (although the variation of overall absorption would be rather less).

Because the attenuation rates, given in **Table D-1**, are arithmetic averages, the complete set cannot be associated with a single reference atmosphere (i.e. with specific values of temperature and relative humidity). They can only thought of as properties of a purely notional atmosphere - referred to as the 'AIR-1845 atmosphere'.

**Table D-1: Average atmospheric attenuation rates used to normalise NPD data in the ANP database.**

| Centre frequency of 1/3-octave band [Hz] | Attenuation rate [dB/100m] | Centre frequency of 1/3-octave band [Hz] | Attenuation rate [dB/100m] |
|---|---|---|---|
| 50 | 0.033 | 800 | 0.459 |
| 63 | 0.033 | 1 000 | 0.590 |
| 80 | 0.033 | 1 250 | 0.754 |
| 100 | 0.066 | 1 600 | 0.983 |
| 125 | 0.066 | 2 000 | 1.311 |
| 160 | 0.098 | 2 500 | 1.705 |
| 200 | 0.131 | 3 150 | 2.295 |
| 250 | 0.131 | 4 000 | 3.115 |
| 315 | 0.197 | 5 000 | 3.607 |
| 400 | 0.230 | 6 300 | 5.246 |
| 500 | 0.295 | 8 000 | 7.213 |
| 630 | 0.361 | 10 000 | 9.836 |

The attenuation coefficients in **Table D-1** may be assumed valid over reasonable ranges of temperature and humidity. However, to check whether adjustments may be necessary, ARP-866A should be used to calculate average atmospheric absorption coefficients for the average airport temperature *T* and relative humidity *RH.* Where, from a comparison of these with those in **Table D-1**, it is judged that adjustment is required the following methodology should be used.

The ANP database provides the following NPD data for each power setting:

- maximum sound level versus slant distance, $L_{max}(d)$
- time integrated level versus distance for the reference airspeed, $L_E(d)$, and
- unweighted reference sound spectrum at a slant distance of 305 m (1000 ft), $L_{n,ref}(d_{ref})$ where $n$ = frequency band (ranging from 1 to 24 for 1/3-octave bands with centre frequencies from 50Hz to 10kHz),

all data being normalised to the AIR-1845 atmosphere.

Adjustment of the NPD curves to user-specified conditions $T$ and $RH$ is performed in three steps:

1. First the reference spectrum is corrected to remove the SAE AIR-1845 atmospheric attenuation $\alpha_{n,ref}$ :

$$L_n(d_{ref}) = L_{n,ref}(d_{ref}) + \alpha_{n,ref} \cdot d_{ref} \qquad \text{(D-1)}$$

   where $L_n(d_{ref})$ is the unattenuated spectrum at $d_{ref}$ = 305m and $\alpha_{n,ref}$ is the coefficient of atmospheric absorption for the frequency band $n$ taken from **Table D-1** (but expressed in dB/m).

2. Next the corrected spectrum is adjusted to each of the ten standard NPD distances $d_i$ using attenuation rates for both (i) the SAE AIR-1845 atmosphere and (ii) the user-specified atmosphere (based on SAE ARP-866A).

   (i) For the SAE AIR-1845 atmosphere:

$$L_{n,ref}(d_i) = L_n(d_{ref}) - 20.\lg(d_i / d_{ref}) - \alpha_{n,ref} \cdot d_i \qquad \text{(D-2)}$$

   (ii) For the user atmosphere:

$$L_{n,866A}(T, RH, d_i) = L_n(d_{ref}) - 20.\lg(d_i / d_{ref}) - \alpha_{n,866A}(T, RH) \cdot d_i \qquad \text{(D-3)}$$

   where $\alpha_{n,866A}$ is the coefficient of atmospheric absorption for the frequency band $n$ (expressed in dB/m) calculated using SAE ARP-866A with temperature $T$, and relative humidity $RH$.

3. At each NPD distance $d_i$ the two spectra are A-weighted and decibel-summed to determine the resulting A-weighted levels $L_{A,866A}$ and $L_{A,ref}$ - which are then subtracted arithmetically:

$$\Delta L(T, RH, d_i) = L_{A,866A} - L_{A,ref} = 10 \cdot \lg \sum_{n=1}^{24} 10^{(L_{n,866A}(T,RH,d_i) - A_n)/10} - 10 \cdot \lg \sum_{n=1}^{24} 10^{(L_{n,ref}(d_i) - A_n)/10} \qquad \text{(D-4)}$$

The increment $\Delta L$ is the difference between the NPDs in the user-specified atmosphere and the reference atmosphere. This is added to the ANP database NPD data value to derive the adjusted NPD data.

Applying $\Delta L$ to adjust both $L_{max}$ and $L_E$ NPDs effectively assumes that different atmospheric conditions affect the reference spectrum only and have no effect on the shape of the level-time-history This may be considered valid for typical propagation ranges and typical atmospheric conditions.

## APPENDIX E: THE FINITE SEGMENT CORRECTION

This appendix outlines the derivation of the finite segment correction and the associated energy fraction algorithm described in section 2.7.19.

### E1 Geometry

The energy fraction algorithm is based on the sound radiation of a 'fourth-power' 90-degree dipole sound source. This has directional characteristics which approximate those of jet aircraft sound, at least in the angular region that most influences sound event levels beneath and to the side of the aircraft flight path.

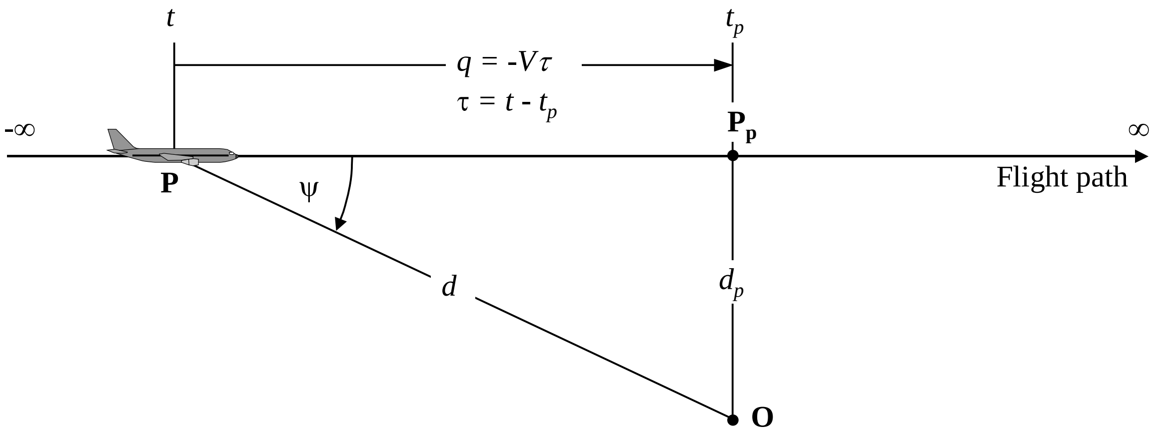


**Figure E-1: Geometry between flight path and observer location O**

**Figure E-1** illustrates the geometry of sound propagation between the flight path and the observer location **O**. The aircraft at **P** is flying in still uniform air with a constant speed on a straight, level flight path. Its closest point of approach to the observer is $\mathbf{P_p}$. The parameters are:

| | |
|---|---|
| $d$ | distance from the observer to the aircraft |
| $d_p$ | perpendicular distance from the observer to the flight path (slant distance) |
| $q$ | distance from **P** to $\mathbf{P_p}$ = $-V\cdot\tau$ |
| $V$ | speed of the aircraft |
| $t$ | time at which the aircraft is at point **P** |
| $t_p$ | time at which the aircraft is located at the point of closest approach $\mathbf{P_p}$ |

$\tau$ flight time = time relative to time at **$P_p$** = t - $t_p$

$\psi$ angle between flight path and aircraft-observer vector

It should be noted that, since the flight time $\tau$ relative to the point of closest approach is negative when the aircraft is before the observer position (as shown in **Figure E-1**), the relative distance *q* to the point of closest approach becomes positive in that case. If the aircraft is ahead of the observer, *q* becomes negative.

## E2 Estimation of the energy fraction

The basic concept of the energy fraction is to express the noise exposure *E* produced at the observer position from a flight path segment **$P_1P_2$** (with a start-point **$P_1$** and an end-point **$P_2$**) by multiplying the exposure $E_\infty$ from the whole infinite path flyby by a simple factor – the *energy fraction* factor *F*:

$$E = F \cdot E_\infty \qquad \text{(E-1)}$$

Since the exposure can be expressed in terms of the time-integral of the mean-square (weighted) sound pressure level, i.e.

$$E = const \cdot \int p^2(\tau) d\tau \qquad \text{(E-2)}$$

to calculate *E* , the mean-square pressure has to be expressed as a function of the known geometric and operational parameters. For a 90° dipole source,

$$p^2 = p_p^2 \cdot \frac{d_p^2}{d^2} \cdot \sin^2 \psi = p_p^2 \cdot \frac{d_p^4}{d^4} \qquad \text{(E-3)}$$

where $p^2$ and $p_p^2$ are the observed mean-square sound pressures produced by the aircraft as it passes points **P** and **$P_p$**.

This relatively simple relationship has been found to provide a good simulation of jet aircraft noise, even though the real mechanisms involved are extremely complex. The term $d_p^2/d^2$ in Equation E-3 describes just the mechanism of spherical spreading appropriate to a point source, an infinite sound speed and a uniform, non-dissipative atmosphere. All other physical effects - source directivity, finite sound speed, atmospheric absorption, Doppler-shift etc. - are implicitly covered by the $sin^2\psi$ term. This factor causes the mean square pressure to decrease inversely as $d^4$; whence the expression “fourth power” source.

Introducing the substitutions

$$d^2 = d_p^2 + q^2 = d_p^2 + (V \cdot \tau)^2 \quad \text{and} \quad \left(\frac{d}{d_p}\right)^2 = 1 + \left(\frac{V \cdot \tau}{d_p}\right)^2$$

the mean-square pressure can be expressed as a function of time (again disregarding sound propagation time):

$$p^2 = p_p^2 \cdot \left(1 + \left(\frac{V \cdot \tau}{d_p}\right)^2\right)^{-2} \qquad \text{(E-4)}$$

Putting this into equation (E-2) and performing the substitution

$$\alpha = \frac{V \cdot \tau}{d_p} \qquad \text{(E-5)}$$

the sound exposure at the observer from the flypast between the time interval $[\tau_1, \tau_2]$ can be expressed as

$$E = const \cdot p_p^2 \cdot \frac{d_p}{V} \cdot \int_{\alpha_1}^{\alpha_2} \frac{1}{(1+\alpha^2)^2} d\alpha \qquad \text{(E-6)}$$

The solution of this integral is:

$$E = const \cdot p_p^2 \cdot \frac{d_p}{V} \cdot \frac{1}{2}\left(\frac{\alpha_2}{1+\alpha_2^2} + \arctan \alpha_2 - \frac{\alpha_1}{1+\alpha_1^2} - \arctan \alpha_1\right) \qquad \text{(E-7)}$$

Integration over the interval $[-\infty,+\infty]$ (i.e. over the whole infinite flight path) yields the following expression for the total exposure $E_\infty$:

$$E_\infty = const \cdot \frac{\pi}{2} \cdot p_p^2 \cdot \frac{d_p}{V} \qquad \text{(E-8)}$$

and hence the energy fraction according to Equation E-1 is

$$F = \frac{1}{\pi}\left(\frac{\alpha_2}{1+\alpha_2^2} + \arctan \alpha_2 - \frac{\alpha_1}{1+\alpha_1^2} - \arctan \alpha_1\right) \qquad \text{(E-9)}$$

## E3 consistency of maximum and time integrated metrics – the scaled distance

A consequence of using the simple dipole model to define the energy fraction is that it implies a specific theoretical difference $\Delta L$ between the event noise levels $L_{max}$ and $L_E$. If the contour model is to be internally consistent, this needs to equal the difference of the values determined from the NPD curves. A problem is that the NPD data are derived from actual aircraft noise measurements - which do not necessarily accord with the simple theory. The theory therefore needs an added element of flexibility. But in principal the variables $\alpha_1$ and $\alpha_2$ are determined by geometry and aircraft speed – thus leaving no further degrees of freedom. A solution is provided by the concept of a *scaled distance* $d_\lambda$ as follows.

The exposure level $L_{E,\infty}$ as tabulated as a function of $d_p$ in the ANP database for a reference speed $V_{ref}$, can be expressed as

$$L_{E,\infty}\left(V_{ref}\right) = 10 \cdot \lg\left[\frac{\int_{-\infty}^{\infty} p^2 . dt}{p_0^2 \cdot t_{ref}}\right] \tag{E-10}$$

where $p_0$ is a standard reference pressure and $t_{ref}$ is a reference time (= 1 s for SEL). For the actual speed $V$ it becomes

$$L_{E,\infty}(V) = L_{E,\infty}\left(V_{ref}\right) + 10 \cdot \lg\left(\frac{V_{ref}}{V}\right) \tag{E-11}$$

Similarly the maximum event level $L_{max}$ can be written

$$L_{max} = 10 \cdot \lg\left[\frac{p_p^2}{p_0^2}\right] \tag{E-12}$$

For the dipole source, using equations E-8, E-11 and E-12, noting that (from equations E-2 and E-8) $\int_{-\infty}^{\infty} p^2 \cdot dt = \frac{\pi}{2} \cdot p_p^2 \cdot \frac{d_p}{V}$ , the difference $\Delta L$ can be written:

$$\Delta L = L_{E,\infty} - L_{max} = 10 \cdot \lg\left[\frac{V}{V_{ref}} \cdot \left(\frac{\pi}{2} p_p^2 \frac{d_p}{V}\right) \cdot \frac{1}{p_0^2 \cdot t_{ref}}\right] - 10 \cdot \lg\left[\frac{p_p^2}{p_0^2}\right] \tag{E-13}$$

This can only be equated to the value of $\Delta L$ determined from the NPD data if the slant distance $d_p$ used to calculate the energy fraction is substituted by a *scaled distance* $d_\lambda$ given by

$$d_\lambda = \frac{2}{\pi} \cdot V_{ref} \cdot t_{ref} \cdot 10^{\left(L_{E,\infty} - L_{max}\right)/10} \tag{E-14a}$$

or

$$d_\lambda = d_0 \cdot 10^{\left(L_{E,\infty} - L_{max}\right)/10} \quad \text{with} \quad d_0 = \frac{2}{\pi} \cdot V_{ref} \cdot t_{ref} \tag{E-14b}$$

Replacing $d_p$ by $d_\lambda$ in equation E-5 and using the definition $q = V\tau$ from **Figure E-1** the parameters $\alpha_1$ and $\alpha_2$ in equation E-9 can be written (putting $q = q_1$ at the start-point and $q - \lambda = q_2$ at the endpoint of a flight path segment of length $\lambda$) as

$$\alpha_1 = \frac{-q_1}{d_\lambda} \qquad \text{and} \qquad \alpha_2 = \frac{-q_1 + \lambda}{d_\lambda} \tag{E-15}$$

Having to replace the slant actual distance by scaled distance diminishes the simplicity of the fourth-power 90 degree dipole model. But as it is effectively calibrated *in situ* using data derived from measurements, the energy fraction algorithm can be regarded as semi-empirical rather than a pure theoretical.